Published by Institute of Physics Publishing for SISSA/ISAS
Received: *July 3, 2003*
Accepted: *October 2, 2003*

BPS action and superpotential for heterotic string compactifications with fluxes

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Abstract: We consider $\mathcal{N} = 1$ compactifications to four dimensions of heterotic string theory in the presence of fluxes. We show that up to order $\mathcal{O}\left(\alpha'^2\right)$ the associated action can be written as a sum of squares of BPS-like quantities. In this way we prove that the equations of motion are solved by backgrounds which fulfill the supersymmetry conditions and the Bianchi identities. We also argue for the expression of the related superpotential and discuss the radial modulus stabilization for a class of examples.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua.

Contents

1. Introduction

One of the main problems of string theory is the large vacuum degeneracy which appears in the effective theory following from compactification. This spoils part of its predictive power and poses the problem of finding some mechanism to remove this degeneracy. A possible way out is provided by considering flux compactifications [1]–[54]. If expectation values are assigned to Neveu-Schwarz and/or Ramond-Ramond form-fields, a potential depending on the moduli is generated in the four-dimensional effective theory and, in addition, there is a backreaction on the geometry, which generically turns the internal space into a non-Kähler manifold. The appearance of a potential is important because it may lead to a fixing of some or all of the moduli, giving us some understanding about vacuum selection. It may also lead to a spontaneous breaking of supersymmetry.

The possibility of introducing background fluxes was already considered in the early studies of supersymmetric compactifications of heterotic string theory [55, 1, 2], but compactifications with background fluxes never received as much attention as those on Calabi-Yau spaces. The main problem with implementing such a scenario is the existence of certain no-go theorems which forbid compactifications with fluxes using arguments following from the analysis of the equations of motion [2, 56] (see also [11] for a different approach). These powerful arguments rule out even non-supersymmetric regular solutions and allow only for the wellknown purely geometric compactifications. These no-go theorems, however, do not apply when singular sources are added to the theory or higher derivative terms are present in the action. This has led to the study of many compactification examples in type IIA/B in the presence of D-branes and orientifold planes [6, 23, 24, 38, 44, 48, 49, 54]. In the heterotic context, on the other hand, the anomaly cancellation requirement leads to a modification of the Bianchi identity of the Neveu-Schwarz three-form and hence to the presence of higher derivative curvature terms in the action. Therefore, and in contrast to type II string theory, the heterotic theory has built into it an interesting mechanism for avoiding the above no-go theorems. Since the heterotic action and the associated supersymmetry rules do not depend on the introduction of sources, the analysis that can be performed in search for flux compactifications has general validity and the resulting classification does not depend on specific choices in the initial setup.

The first analysis of the heterotic theory with fluxes [55, 1, 2] led to a set of necessary and sufficient conditions for obtaining supersymmetric solutions to first order in the α' expansion. Despite the formal simplicity of the resulting geometrical conditions, explicit solutions preserving the minimal amount of supersymmetry are still lacking today, after more than fifteen years. The only examples discussed in the literature so far were obtained by applying duality transformations to type-II or M-theory solutions [6, 33, 38, 44, 45]. However, it has not been shown explicitly that these configurations solve the heterotic Bianchi identity for the Neveu-Schwarz three-form field, and although these solutions may be exact solutions to the full heterotic theory, their intrinsic non-perturbativeness (at least in α') makes it unlikely that they will satisfy the geometrical conditions obtained in [1], since the latter, which were derived by truncating the theory to first order in α', receive higher order corrections in α'.

A different approach to explicitly construct and classify new examples of heterotic backgrounds in the presence of fluxes is given by the use of the torsion classification for SU(3) structures on six-dimensional manifolds [57]. The use of the structure group of the tangent bundle provides an organizing principle for solutions with fluxes similar to the notion of holonomy group for the purely geometric compactifications. Such a tool can lead to an easy classification and construction of solutions to the geometric conditions coming from the supersymmetry rules[1] [28, 37]. Unfortunately, in order to really generate a complete solution of the heterotic theory one also needs to solve the Bianchi identities for the various fields, and this has to be worked out explicitly on a case by case basis, in contrast to what happens for Calabi-Yau backgrounds.

Solving the residual supersymmetry equations of [1] does not immediately guarantee that also the equations of motion are satisfied. In this note we address this issue by showing that the action of the heterotic theory (truncated to second order in the α' expansion) can be rewritten as a sum of BPS-like squares. This then implies that solutions to the equations of motion can be found by setting these BPS-like squares to zero. An interesting outcome of this rewriting is that the conditions appearing in the squares are precisely the geometric conditions discussed in [1, 37, 46]. Therefore, the heterotic equations of motion (which have contributions from higher curvature terms) do not impose additional constraints on the solutions constructed by solving the supersymmetry equations of [1] and the Bianchi identities for the various fields.

Integrating the action over the internal manifold yields the potential appearing in the four-dimensional effective $\mathcal{N} = 1$ supergravity theory. This potential can be rewritten in

[1] Group structures as a tool to classify and construct new solutions to supergravity and string theory were first used in [28] and then also in [29, 36, 37, 39, 42, 43, 46, 50] in connection with flux compactifications.

terms of a superpotential W and various D-terms. The necessary requirements for having an $\mathcal{N}=1$ vacuum, i.e. $W=0$ and $\partial W=0$, then impose certain conditions leading to moduli stabilization. A rigorous derivation of the superpotential for flux compactifications requires a detailed knowledge of the moduli space of the compactification manifolds, which is not available at present. An educated guess for the superpotential,[2] which we will argue for, is given by

$$W=\int\left(H+\frac{i}{2}dJ\right)\wedge\Omega\,. \tag{1.1}$$

Notice that for generic flux compactifications the internal space is not complex, i.e. J is not integrable, and therefore $dJ\wedge\Omega\neq 0$. Since the above superpotential explicitly depends on the three-form flux H, its extremisation should give rise to the torsional constraints leading to supersymmetric configurations. Again, a rigorous derivation requires an explicit knowledge of the metric moduli, but we will argue that under certain assumptions the expected torsional constraints do follow. More precisely, the superpotential W has to lead to a determination of H in terms of the deviation of the internal space from being a Calabi-Yau manifold. W must therefore also include pieces which are purely geometrical and which measure the non-Calabi-Yau-ness of the internal space. This is captured by the additional piece proportional to dJ in (1.1).

In the final section of the paper we will discuss the issue of the radial modulus stabilization for a class of examples prominently used in the literature. We will argue that, on generic grounds, twisted tori cannot be used as consistent supergravity solutions of heterotic compactifications with fluxes, at least on the basis of an order by order α' expansion. We will finish with some comments on higher order α' corrections to the conditions on the geometry of the internal space.

2. BPS rewriting of the action

As stated in the introduction, the main results obtained so far on heterotic compactifications with fluxes follow from the supersymmetry equations [1]. A solution to these equations does, however, not guarantee that the equations of motion are also satisfied. A nice way to find solutions to the equations of motion consists in rewriting the action in terms of a sum of squares. In this way, by making all the squares vanish, one also ensures the vanishing of their variation, and therefore the action is extremised. The resulting solutions are then described in terms of simpler equations which often coincide with those needed to obtain supersymmetric backgrounds. Generically, however, one cannot expect that the equations of motion do not impose additional constraints beyond those following from the requirement of supersymmetry. Here we are going to show that solutions to the heterotic supersymmetry equations do satisfy the heterotic equations of motion, and we do this by rewriting the heterotic action in terms of squares (for another approach leading to a similar conclusion see [53]).

[2] The superpotential (1.1) and its origin were also discussed recently in [52] from an alternative point of view.

In order to show this, we will first review the conditions following from the requirement of preserving $\mathcal{N}=1$ supersymmetry in four dimensions. Then we will show how the action of the heterotic theory truncated to second order in the α' expansion can be rewritten in terms of BPS-like squares, and finally we will show that from their vanishing one recovers the geometrical conditions following from the supersymmetry conditions.

2.1 Supersymmetry conditions

Necessary and sufficient conditions for $\mathcal{N}=1$ spacetime supersymmetry in four dimensions from the heterotic string supersymmetry rules to first order in α' were derived in [1]. The metric in the string frame was assumed to be the warped product of a four-dimensional maximally symmetric space-time and a six-dimensional compact manifold,

$$ds^2 = g_{MN}\, dx^M \otimes dx^N = \mathrm{e}^{2(\Delta(y)-\phi(y))} \left(dx^\mu \otimes dx^\nu\, \hat{g}_{\mu\nu}(x) + dy^m \otimes dy^n\, \hat{g}_{mn}(y)\right). \qquad (2.1)$$

The relevant fermionic supersymmetry rules are given by

$$\delta\psi_M = \nabla^-_M \epsilon \equiv \nabla_M \epsilon - \frac{1}{4} H_{MNP}\,\Gamma^{NP}\epsilon\,, \qquad (2.2)$$

$$\delta\chi = -\frac{1}{4}\Gamma^{MN}\epsilon\, F_{MN}\,, \qquad (2.3)$$

$$\delta\lambda = \nabla\!\!\!\!/\,\phi\epsilon + \frac{1}{24}\Gamma^{MNP} H_{MNP}\,\epsilon\,. \qquad (2.4)$$

The vanishing of the supersymmetry equations forces the four-dimensional spacetime to be minkowskian, i.e. $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$. For what concerns the internal six-dimensional manifold, supersymmetry requires it to be complex, of SU(3) holonomy with respect to the special connection with torsion ∇^- and admitting one nowhere vanishing globally defined holomorphic (3,0)-form Ω. The norm of the latter must also be related to the complex structure J,

$$\star d \star J = i\left(\bar{\partial} - \partial\right) \log ||\Omega||\,. \qquad (2.5)$$

All the ten-dimensional fields are expressible in terms of the geometry. The dilaton ϕ is related to the warp factor Δ and both are determined by the norm of the holomorphic (3,0)-form,

$$\Delta(y) = \phi(y) = \frac{1}{8}\log||\Omega|| + \text{constant}\,. \qquad (2.6)$$

The three-form H expectation value is determined in terms of the complex structure,[3]

$$H = \frac{i}{2}\left(\partial - \bar{\partial}\right) J\,. \qquad (2.7)$$

Finally, the Yang-Mills background field strength must be a (1,1)-form satisfying $F_{mn}J^{mn} = 0$. On top of this, one must satisfy the Bianchi identities for the various forms.

[3] Equations (2.5) and (2.7) differ by a sign from the ones presented in [1].

These geometric constraints can be translated into conditions on the intrinsic torsion τ of the Levi-Civita connection of the compactification manifold. It has been shown [37, 46] that the torsion must lie in

$$\tau \in \mathcal{W}_3 \oplus \mathcal{W}_4 \oplus \mathcal{W}_5 \,, \tag{2.8}$$

that the last two are related by

$$2\,\mathcal{W}_4 + \mathcal{W}_5 = 0 \,, \tag{2.9}$$

and that

$$\mathcal{W}_4 \quad \text{and} \quad \mathcal{W}_5 \qquad \text{exact and real} \,. \tag{2.10}$$

Here $\mathcal{W}_1 \ldots \mathcal{W}_5$ denote the 5 different classes of irreducible modules in which the torsion can be decomposed. The various classes can be read off from dJ and $d\psi$, where J is the two-form associated to the almost complex structure and ψ is a nowhere vanishing three-form,

$$dJ = \frac{3}{4}\, i \left(\mathcal{W}_1\, \overline{\psi} - \overline{\mathcal{W}}_1\, \psi\right) + \mathcal{W}_3 + J \wedge \mathcal{W}_4 \,, \tag{2.11}$$

$$d\psi = \mathcal{W}_1 J \wedge J + J \wedge \mathcal{W}_2 + \psi \wedge \mathcal{W}_5 \,, \tag{2.12}$$

where $J \wedge \mathcal{W}_3 = J \wedge J \wedge \mathcal{W}_2 = 0$ and $\psi \wedge \mathcal{W}_3 = 0$.

2.2 Rewriting of the action

Let us now rewrite the action of the heterotic theory in terms of BPS-like squares. The bosonic part of the Lagrangean up to second order in α' is given by [68]

$$\begin{aligned} S = \int d^{10}x\, \sqrt{g}\, e^{8\phi} \Big[\frac{1}{4} R - \frac{1}{12} H_{MNP} H^{MNP} + 16 (\partial_M \phi)^2 - \\ - \frac{1}{4} \alpha' \left(F^I_{MN} F^{I\,MN} - R^{+}_{MNPQ} R^{+\,MNPQ} \right) \Big] \,. \end{aligned} \tag{2.13}$$

This action is written in the string frame and its fermionic completion makes it supersymmetric using the three-form Bianchi identity given by

$$dH = \alpha' \left(\mathrm{tr}\, R^+ \wedge R^+ - \mathrm{tr}\, F \wedge F \right) , \tag{2.14}$$

where the curvature R^+ is the generalized Riemann curvature built from the generalized connection ∇^+ (see the appendix). This Bianchi identity is of particular interest, because it imposes an interplay between terms of different orders in α'. The α' corrections to the standard Bianchi identity for the three-form field are needed to cancel the Lorentz and gauge anomalies which are present in the quantum theory of type-I supergravity in ten-dimensions coupled to Yang-Mills fields, which is the low-energy limit of the heterotic string. These anomalies can be cancelled by the Green-Schwarz mechanism, which implies the modification of the three-form Bianchi identity by α' terms proportional to the first Pontrjagin forms for the gauge and tangent bundles. This cancellation still works for any choice of the torsion tensor used in the definition of the generalized Riemann curvature appearing in the Bianchi identity [58]. Of course, one does not expect all these choices to have a different physical meaning and indeed they may be related by field redefinitions [59]. Once such a choice has been made, though, the supersymmetrization of the Bianchi identity (to a given order in the α' expansion) imposes a specific form of the associated low-energy equations of motion and action. The equations of motion for the bosonic fields for some of these possible choices and for various α' orders have been computed using different approaches [60]–[68]. For the purpose of rewriting the action in terms of squares of the supersymmetry conditions discussed above, we have used the approach presented in [68] and rescaled the various fields according to the conventions of [1].

In this setup the generalized connections defining the covariantly constant supersymmetry spinor (2.2) and the one used in the H-Bianchi identity (2.14) are different. This choice is not the same as the one previously used in [37], but it turns out to be convenient for the purpose of our calculation. As discussed above, a different choice would have been equally valid. One would have just needed to compute again the supersymmetric completion of the action. The α' corrections would have been different, possibly containing different combinations of the three-form field than just the one appearing in R^+. Note that the leading higher-order terms in the Riemann curvature in (2.13) do not appear in the characteristic form of the Gauss-Bonnet combination, though this combination may be generated by a vielbein redefinition [68].

In the search for a BPS rewriting of (2.13), we will now use the metric ansatz (2.1) and require the internal space to admit an SU(3) structure. This means that such a manifold allows for a globally defined orthogonal almost complex structure,

$$J_m{}^p J_p{}^n = -\delta_m^n \,, \qquad J_m{}^p J_n{}^q g_{pq} = g_{mn} \,, \tag{2.15}$$

and a complex three-form ψ, which is of $(3,0)$-type with respect to J. In order to consistently obtain that setting to zero the BPS-like squares implies a solution to the equations of motion, we also impose that the only degrees of freedom for the various fields are given by expectation values on the internal space and are functions only of the internal coordinates.

To simplify the discussion we limit ourselves to the case with dilaton and warp factor identified, i.e. $\phi = \Delta$, but the generalization of the following results is straightforward. After some manipulations, which we are going to describe in the next subsection, the action (2.13) can be written as

$$\begin{aligned} S = \int d^4x\sqrt{g_4}\Bigg\{ &-\frac{1}{2}\int_{\mathcal{M}_6} \mathrm{e}^{8\phi}\,(8d\phi+\theta)\wedge\star\,(8d\phi+\theta) + \frac{1}{8}\int_{\mathcal{M}_6} \mathrm{e}^{8\phi} J\wedge J\wedge \hat{R}^{ab}J_{ab} - \\ &- \frac{1}{4}\int d^6y\,\sqrt{g_6}\,\mathrm{e}^{8\phi} N_{mn}{}^p g^{mq} g^{nr} g_{ps} N_{qr}{}^s + \\ &+ \frac{1}{2}\int_{\mathcal{M}_6} \mathrm{e}^{8\phi}\left(H + \frac{1}{2}\star\mathrm{e}^{-8\phi} d(\mathrm{e}^{8\phi}J)\right)\wedge\star\left(H + \frac{1}{2}\star\mathrm{e}^{-8\phi} d(\mathrm{e}^{8\phi}J)\right) - \\ &- \frac{\alpha'}{2}\int d^6y\sqrt{g_6}\,\mathrm{e}^{8\phi}\left[\mathrm{tr}(F^{(2,0)})^2 + \mathrm{tr}(F^{(0,2)})^2 + \frac{1}{4}\,\mathrm{tr}(J^{mn}F_{mn})^2\right] + \\ &+ \frac{\alpha'}{2}\int d^6y\sqrt{g_6}\,\mathrm{e}^{8\phi}\left[\mathrm{tr}(R^{+(2,0)})^2 + \mathrm{tr}(R^{+(0,2)})^2 + \frac{1}{4}\,\mathrm{tr}(J^{mn}R^{+}_{mn})^2\right]\Bigg\}\,. \end{aligned} \tag{2.16}$$

In this expression the traces are taken with respect to the fiber indices $a, b, \ldots$, whereas the Hodge type refers to the base indices $m, n, \ldots$ of the curvatures. The other geometrical objects appearing in the above expression are the Lee-form

$$\theta \equiv J \lrcorner dJ = \frac{3}{2} J^{mn}\, \partial_{[m} J_{np]}\, dx^p\,, \tag{2.17}$$

the Nijenhuis tensor

$$N_{mn}{}^p = J_m{}^q \partial_{[q} J_{n]}{}^p - J_n{}^q \partial_{[q} J_{m]}{}^p\,, \tag{2.18}$$

and the generalized curvature $\hat{R}$, which is constructed using the Bismut connection built from the standard Levi-Civita connection and a totally antisymmetric torsion T^B proportional to the complex structure,

$$T^B_{mnp} = \frac{3}{2}\, J_m{}^q J_n{}^r J_p{}^s \partial_{[q} J_{rs]} = -\frac{3}{2} J_{[m}{}^q \nabla_{|q|} J_{np]}\,. \tag{2.19}$$

The action (2.16) will now be used to find the conditions determining the background geometry. If, on the other hand, one also wants to consider fluctuations around these backgrounds then (2.16) can be used as the expression for the scalar potential of the effective four-dimensional theory,

$$S = -\int d^4x\, \sqrt{g_4}\, V\,. \tag{2.20}$$

The action (2.16) consists of a sum of squares as well as of one linear term. In order to have a solution of the equations of motion one sets to zero all the squares and proves that the linear term does not contribute to the equations of motion. We will come back to the linear term in a moment, but first we would like to exhibit the correspondence between the supersymmetry conditions and the terms which are squared in the action. The geometrical conditions resulting from the vanishing of the BPS-like squares are the vanishing of the Nijenhuis tensor

$$N^m{}_{np} = 0$$

and of some components of the generalized Riemann curvature constructed from the ∇^+ connection,

$$R^{+\,(2,0)} = R^{+\,(0,2)} = J^{mn} R^+_{mn} = 0.$$

The vanishing of the Nijenhuis tensor states that the internal manifold is complex (which means $\mathcal{W}_1 = \mathcal{W}_2 = 0$ in the torsion classes language). The conditions on the R^+ curvature can be translated into the integrability constraints following from the vanishing of the gravitino supersymmetry transformation (2.2), which leads to the requirement of SU(3) holonomy for the ∇^- connection. The proof requires the identity

$$R^+_{ab\,cd} = R^-_{cd\,ab} - (dH)_{abcd}\,, \tag{2.21}$$

which relates the R^+ and R^- curvatures with the base and fiber indices swapped. Using this identity and the fact that dH gives higher order terms in α' the conditions on the base indices of R^+ become conditions on the R^- fiber indices, to lowest order in α',

$$R^{-\,(2,0)} = R^{-\,(0,2)} = J^{ab} R^-_{ab} = 0\,. \tag{2.22}$$

These conditions precisely state that the generalized curvature R^- is in the adjoint representation of SU(3) $\subset$ SO(6) and therefore its holonomy group is contained in SU(3). We also obtain the relation between the $\mathcal{W}_4$ and $\mathcal{W}_5$ torsion classes, expressed by the identification of the differential of the dilaton with the Lee-form,

$$d\phi + \frac{1}{8}\theta = 0 \,, \tag{2.23}$$

as follows from (2.5) and (2.6). The conditions in the gauge sector are also the same conditions one gets from the supersymmetry variation of the gaugino, i.e. the gauge field strength is of type $(1,1)$ and J traceless. The final BPS-like square precisely yields the locking condition of the three-form H onto the almost complex structure J which was derived from the vanishing of the supersymmetry transformations. Indeed, on a complex manifold, and using (2.23), the following identity holds [69],

$$H = -\frac{1}{2} \star \mathrm{e}^{-8\phi}\, d(\mathrm{e}^{8\phi}\, J) = \frac{1}{2} i(\partial - \bar{\partial}) J \,. \tag{2.24}$$

Let us now show that the linear term does not give any further condition. The locking condition (2.24) implies the identification of the connection ∇^- defining the generalized curvature R^- with the Bismut connection defining the generalized curvature $\hat{R}$. In this way, on the solution, the remaining linear term can also be rewritten as

$$\int \mathrm{e}^{8\phi}\, J \wedge J \wedge \hat{R}^{ab} J_{ab} = 2 \int \mathrm{e}^{8\phi} \star J \wedge c_1\left(R^-\right) . \tag{2.25}$$

The Ricci two-form $c_1\left(R^-\right)$ vanishes according to (2.22) and so does the linear term. Now consider varying (2.25) with respect to the metric, i.e.

$$\delta \int \mathrm{e}^{8\phi}\, J \wedge J \wedge \hat{R}^{ab} J_{ab} = 2 \int \mathrm{e}^{8\phi} \star J \wedge \delta c_1(\hat{R}) \,, \tag{2.26}$$

where we used that $c_1(\hat{R}) = c_1(R^-) = 0$ on the solution. The Ricci two-form of the Bismut connection is the same as the Ricci form of the Chern connection up to an exact form [70, 13],

$$c_1(\hat{R}) = c_1(\mathrm{Chern}) - 2\, d\, (\star d \star J) \,, \tag{2.27}$$

where the Chern connection is defined as

$$C^m{}_{np} = \Gamma^m{}_{np} + \frac{3}{2} J_n{}^q\, \partial_{[q} J_{rp]}\, g^{rm} \,. \tag{2.28}$$

The variation of $c_1(\mathrm{Chern})$, when evaluated on a manifold which is complex, yields

$$\delta\, c_1(\mathrm{Chern})\Big|_{N_{mn}{}^p=0} = \frac{1}{2} d\left[J \cdot d\left(g^{mn} \delta g_{mn}\right)\right] , \tag{2.29}$$

where $J \cdot d = dx^m J_m{}^n \partial_n$. This implies that

$$\delta c_1(\hat{R})\Big|_{N_{mn}{}^p=0} = d\Lambda \,, \tag{2.30}$$

for a certain one-form Λ. Hence the variation (2.26) is vanishing once an integration by parts is performed,

$$\int e^{8\phi} \star J \wedge \delta c_1(\hat{R}) = \int e^{8\phi} \star J \wedge d\Lambda = -\int d\left(e^{8\phi} \star J\right) \wedge \Lambda = 0\,, \qquad (2.31)$$

and once one uses (2.23).

Before proceeding with the derivation of (2.16) let us briefly comment on the limiting case with constant dilaton ϕ = const. and vanishing flux $H = 0$. The locking condition (2.24) simply becomes the requirement for the internal manifold to be Kähler, imposing $dJ = 0$. In addition, the square involving the dilaton becomes a condition imposing the vanishing of the Lee form, $d \star J = 0$. Moreover, now $\nabla^{\pm} = \nabla$ and $R^{+} = R^{-} = R$. Therefore the conditions on the holonomy of ∇^{-} become conditions on the Levi-Civita connection. The solution is obviously given by Calabi-Yau manifolds, which are Kähler and have vanishing first Chern class.

2.3 Details of the derivation

We now discuss the derivation of the action (2.16) in detail. We will see that several interesting geometrical relations arise and that some non-trivial cancellations occur.

The first interesting and quite non-trivial relation emerges when one tries to express the Ricci scalar of the Levi-Civita connection in terms of other geometric quantities of the internal manifold. After a tedious calculation one obtains that

$$\begin{aligned}\int d^6y\,\sqrt{g_6}\,e^{8\phi}R = &-\int e^{8\phi}\theta\wedge\star\theta + \frac{1}{2}\int e^{8\phi}J\wedge J\wedge R^{ab}J_{ab} - 16\int e^{8\phi}d\phi\wedge\star\theta - \\ &-2\int e^{8\phi}dJ^{(3,0)+(0,3)}\wedge\star dJ^{(3,0)+(0,3)} - \\ &-\int d^6y\,\sqrt{g_6}\,e^{8\phi}N_{mn}{}^{p}g^{mq}g^{nr}g_{ps}N_{qr}{}^{s}\,, \end{aligned} \qquad (2.32)$$

where total derivatives were dropped. Therefore the Ricci scalar has been related to the square of the Lee form θ, the Nijenhuis tensor $N_{mn}{}^{p}$, the $(3,0) + (0,3)$ part of the dJ tensor, the double J trace of the Riemann tensor $R_{ab\,cd}J^{ab}J^{cd}$ and the mixed term between $d\phi$ and θ which we need as a double product in the square relating them.

An obvious manipulation which has to be performed is the rewriting of the vector kinetic term as squares of the conditions following from the gaugino supersymmetry transformations. To this end we use the following identity,

$$\begin{aligned}\int d^6y\sqrt{g}\,e^{8\phi}\,\mathrm{tr}(F_{mn}F^{mn}) \;=\; &+2\int d^6y\sqrt{g}\,e^{8\phi}\left[\mathrm{tr}(F^{(2,0)})^2 + \mathrm{tr}(F^{(0,2)})^2 + \frac{1}{4}\,\mathrm{tr}(J^{mn}F_{mn})^2\right] - \\ &-2\int e^{8\phi}J\wedge\mathrm{tr}(F\wedge F)\,. \end{aligned} \qquad (2.33)$$

The same relation can also be applied to the higher order term in the Riemann curvature appearing in (2.13), which can be rewritten as

$$\begin{aligned}\int d^6y\,\sqrt{g_6}\,\mathrm{e}^{8\phi}R^+_{mnab}R^{+\,mnab} &= \int d^6y\,\sqrt{g_6}\,\mathrm{e}^{8\phi}\,\mathrm{tr}(R^+_{mn}R^{+mn})\\ &= +2\int d^6y\sqrt{g}\,\mathrm{e}^{8\phi}\Big[\mathrm{tr}(R^{+(2,0)})^2+\mathrm{tr}(R^{+(0,2)})^2+\\ &\qquad\qquad +\frac{1}{4}\,\mathrm{tr}(J^{mn}R^+_{mn})^2\Big]-\\ &\quad -2\int \mathrm{e}^{8\phi}J\wedge\mathrm{tr}(R^+\wedge R^+)\,. \end{aligned}\tag{2.34}$$

We can now attempt to perform the first rewriting of the action (2.13) using (2.32), (2.33) and (2.34),

$$\begin{aligned}S = \int d^4x\,\sqrt{g_4}\Big\{&-\frac{1}{4}\int \mathrm{e}^{8\phi}\,(8d\phi+\theta)\wedge\star(8d\phi+\theta)+\\ &+\frac{1}{8}\int \mathrm{e}^{8\phi}J\wedge J\wedge R^{ab}J_{ab}-\frac{1}{2}\int \mathrm{e}^{8\phi}\star H\wedge H-\\ &-\frac{1}{2}\int \mathrm{e}^{8\phi}dJ^{(3,0)+(0,3)}\wedge\star dJ^{(3,0)+(0,3)}-\\ &-\frac{1}{4}\int d^6y\,\sqrt{g}\mathrm{e}^{8\phi}N_{mn}{}^{p}g^{mq}g^{nr}g_{ps}N_{qr}{}^{s}+\\ &+\frac{\alpha'}{2}\int \mathrm{e}^{8\phi}J\wedge\big[-\mathrm{tr}(R^+\wedge R^+)+\mathrm{tr}(F\wedge F)\big]-\\ &-\frac{\alpha'}{2}\int d^6y\,\sqrt{g_6}\mathrm{e}^{8\phi}\Big[\mathrm{tr}(F^{(2,0)})^2+tr(F^{(0,2)})^2+\frac{1}{4}\,\mathrm{tr}(J^{mn}F_{mn})^2\Big]+\\ &+\frac{\alpha'}{2}\int d^6y\sqrt{g_6}\mathrm{e}^{8\phi}\Big[\mathrm{tr}(R^{+(2,0)})^2+\mathrm{tr}(R^{+(0,2)})^2+\frac{1}{4}\,\mathrm{tr}(J^{mn}R^+_{mn})^2\Big]\Big\}\,. \end{aligned}\tag{2.35}$$

In order to obtain the relations coming from the analysis of the supersymmetry transformation laws it is now more natural to introduce the generalized Riemann curvature $\hat{R}$ constructed from the Bismut connection. An explicit computation shows that

$$\begin{aligned}\int \mathrm{e}^{8\phi}J\wedge J\wedge R_{ab}J^{ab} &= \int \mathrm{e}^{8\phi}J\wedge J\wedge \hat{R}_{ab}J^{ab}+2\int \mathrm{e}^{-8\phi}\Big[d(\mathrm{e}^{8\phi}J)\wedge\star d(\mathrm{e}^{8\phi}J)\Big]+\\ &\quad+\frac{1}{2}\int d^6y\,\sqrt{g_6}\mathrm{e}^{-8\phi}\Big[\star d(\mathrm{e}^{8\phi}J)\Big]_{mnp}J^{mq}J^{nr}\Big[\star d(\mathrm{e}^{8\phi}J)\Big]_{qr}{}^{p}-\\ &\quad-2\int \mathrm{e}^{8\phi}\,(8d\phi+\theta)\wedge\star(8d\phi+\theta)\,, \end{aligned}\tag{2.36}$$

where we performed an integration by parts on the second term and where we discarded total derivatives. Using (2.36) and the three-form Bianchi identity (2.14), some of the terms in (2.35) may be rearranged as

$$\begin{aligned}&\frac{1}{8}\int \mathrm{e}^{8\phi}J\wedge J\wedge R^{ab}J_{ab}-\frac{1}{2}\int \mathrm{e}^{8\phi}\star H\wedge H-\frac{1}{2}\int \mathrm{e}^{8\phi}J\wedge dH=\\ &\quad=\frac{1}{8}\int \mathrm{e}^{8\phi}J\wedge J\wedge\hat{R}^{ab}J_{ab}+\end{aligned}$$

$$+\frac{1}{2}\int \mathrm{e}^{8\phi}\left(H+\frac{1}{2}\star\mathrm{e}^{-8\phi}d(\mathrm{e}^{8\phi}J)\right)\wedge\star\left(H+\frac{1}{2}\star\mathrm{e}^{-8\phi}d(\mathrm{e}^{8\phi}J)\right)-$$
$$-\frac{1}{4}\int \mathrm{e}^{8\phi}\,(8d\phi+\theta)\wedge\star(8d\phi+\theta)+\frac{1}{8}\int \mathrm{e}^{-8\phi}\left[d(\mathrm{e}^{8\phi}J)\wedge\star d(\mathrm{e}^{8\phi}J)\right]+$$
$$+\frac{1}{16}\int\sqrt{g_6}\mathrm{e}^{-8\phi}\left[\star d(\mathrm{e}^{8\phi}J)\right]_{mnp}J^{mq}J^{nr}\left[\star d(\mathrm{e}^{8\phi}J)\right]_{qr}{}^{p}\,, \tag{2.37}$$

where we performed an integration by parts on the $J\wedge dH$ term in order to reproduce the square which gives the H-locking condition. The last line of this expression is of a special form which selects a specific projection of the three-form dJ. Using the projection relations (which we review in the appendix) it can be shown that for a given three-form T

$$4\int\star(T^{(3,0)}+T^{(0,3)})\wedge(T^{(3,0)}+T^{(0,3)})=\int\star T\wedge T-\frac{1}{2}\int d^6y\sqrt{g_6}\,T_{mnp}J^{mq}J^{nr}T_{qr}{}^{p}\,, \tag{2.38}$$

which, upon identification of T with $-\frac{1}{2}\mathrm{e}^{-8\phi}\star d(\mathrm{e}^{8\phi}J)$, yields a new expression for the last line in (2.37).

Inserting now (2.37) and (2.38) into (2.35) and applying all due simplifications one obtains the following expression for the action,

$$\begin{aligned}S=\int d^4x\sqrt{g_4}\Big\{&-\frac{1}{2}\int \mathrm{e}^{8\phi}\,(8d\phi+\theta)\wedge\star(8d\phi+\theta)+\\
&+\frac{1}{8}\int J\wedge J\wedge\hat{R}^{ab}J_{ab}-\frac{1}{4}\int d^6y\,\sqrt{g}\mathrm{e}^{8\phi}N_{mn}{}^{p}g^{mq}g^{nr}g_{ps}N_{qr}{}^{s}+\\
&+\frac{1}{2}\int \mathrm{e}^{8\phi}\left(H+\frac{1}{2}\star\mathrm{e}^{-8\phi}d(\mathrm{e}^{8\phi}J)\right)\wedge\star\left(H+\frac{1}{2}\star\mathrm{e}^{-8\phi}d(\mathrm{e}^{8\phi}J)\right)-\\
&-\frac{1}{2}\int \mathrm{e}^{8\phi}\star(\star dJ)^{(3,0)+(0,3)}\wedge(\star dJ)^{(3,0)+(0,3)}-\\
&-\frac{1}{2}\int \mathrm{e}^{8\phi}dJ^{(3,0)+(0,3)}\wedge\star dJ^{(3,0)+(0,3)}-\\
&-\frac{\alpha'}{2}\int d^6y\,\sqrt{g_6}\mathrm{e}^{8\phi}\left[\mathrm{tr}(F^{(2,0)})^2+\mathrm{tr}(F^{(0,2)})^2+\frac{1}{4}\mathrm{tr}(J^{mn}F_{mn})^2\right]+\\
&+\frac{\alpha'}{2}\int d^6y\,\sqrt{g_6}\mathrm{e}^{8\phi}\left[\mathrm{tr}(R^{+(2,0)})^2+\mathrm{tr}(R^{+(0,2)})^2+\frac{1}{4}\mathrm{tr}(J^{mn}R^{+}_{mn})^2\right]\Big\}\,.\end{aligned} \tag{2.39}$$

The terms proportional to $dJ^{(3,0)+(0,3)}$ cancel out because $\star dJ^{(3,0)+(0,3)}=(\star dJ)^{(3,0)+(0,3)}$, which follows from the projector property

$$P_a{}^iP_b{}^jP_c{}^k\epsilon_{ijkdef}=\epsilon_{abcijk}Q_d{}^iQ_e{}^jQ_f{}^k\,. \tag{2.40}$$

Consequently one obtains the result (2.16) for the action.

3. The superpotential and the torsional constraints

In this section we want to address the problem of deriving the torsional constraints and the potential of the effective $\mathcal{N}=1$ four-dimensional theory in terms of a superpotential

W. We will not present a rigorous proof but rather try to work in close analogy with the type-IIB theory, pointing out the various problems that arise in doing so.

In the case of type-IIB compactifications with 3-form fluxes

$$G = H_R^{(3)} - \tau H_{NS}^{(3)}\,, \tag{3.1}$$

the potential of the effective theory is given by the sum of two terms depending on the warp factor A [16, 32]

$$V = \frac{1}{2\kappa_{10}\,\mathrm{Im}\,\tau}\int e^{4A} G^+ \wedge \star\overline{G^+} + \frac{i}{4\kappa_{10}\,\mathrm{Im}\,\tau}\int e^{4A} G\wedge \bar{G}\,, \tag{3.2}$$

where the first term is an F-term whereas the second term is a topological D-term. In (3.2) the complex three-form G was split into its self- and anti-selfdual parts

$$G = G^+ + G^-\,, \tag{3.3}$$

where

$$\star G^+ = +iG^+\,, \qquad \star G^- = -iG^-\,. \tag{3.4}$$

As expected, the F-term of the potential can be derived from a superpotential which is given by [7, 8, 9, 16, 32]

$$W = \int G\wedge\Omega = \int (H_R^{(3)} - \tau H_{NS}^{(3)})\wedge\Omega\,, \tag{3.5}$$

and its extremization yields the conditions on the fluxes to obtain a supersymmetric Minkowski vacuum.

Let us now try to understand how far such a derivation can be extended to the heterotic theory with fluxes. We saw in the rewriting of the effective action in terms of BPS-like squares (2.16) that there are several terms contributing to the final potential. These terms have different sources and nature and are related to both the introduction of fluxes and to the modification of the geometry. Of course we would like to understand the precise interpretation of some of these terms as F- or D-terms in the construction of the $\mathcal{N}=1$ effective theory. As a first step in this direction we will simply consider the terms involving the H-flux contribution, namely

$$V = -\frac{1}{2}\int \mathrm{e}^{8\phi}\left(H + \frac{1}{2}\star \mathrm{e}^{-8\phi}d(\mathrm{e}^{8\phi}J)\right)\wedge\star\left(H + \frac{1}{2}\star \mathrm{e}^{-8\phi}d(\mathrm{e}^{8\phi}J)\right). \tag{3.6}$$

In trying to push the analogy with the type-IIB case one can introduce the complex three-form

$$\mathcal{H} \equiv H + \frac{i}{2}\mathrm{e}^{-8\phi}d(\mathrm{e}^{8\phi}J)\,, \tag{3.7}$$

which, as we will argue below, plays the same role as the complex three-form G on the type-IIB side. The potential V given in (3.6) can be rewritten in terms of the three-form $\mathcal{H}$ as

$$V = -\frac{1}{2}\left(||\mathrm{e}^{4\phi}\mathcal{H}||^2 - i\int \mathrm{e}^{8\phi}\mathcal{H}\wedge\bar{\mathcal{H}}\right), \tag{3.8}$$

where we introduced the inner product[4] of three-forms on a complex manifold X

$$\langle \alpha_1, \alpha_2 \rangle \equiv \int_X \alpha_1 \wedge \star \bar{\alpha}_2 \,, \tag{3.9}$$

and the norm is simply the inner product of a form with itself.

In perfect analogy with the IIB case one can split the complex three-form $\mathcal{H}$ in terms of its self- and anti-selfdual parts

$$\mathcal{H} = \mathcal{H}^+ + \mathcal{H}^- \,, \tag{3.10}$$

where

$$\star \mathcal{H}^+ = +i\mathcal{H}^+ \,, \qquad \star \mathcal{H}^- = -i\mathcal{H}^- \,. \tag{3.11}$$

After this decomposition is performed the potential (3.8) further simplifies to

$$V = - \int \mathrm{e}^{8\phi} \mathcal{H}^+ \wedge \star \overline{\mathcal{H}^+} \,. \tag{3.12}$$

This is formally the same as the first term in (3.2). Note that there is no analogue of the topological term. This is in contrast with the IIB case, where such a term was crucially involved in the tadpole cancellation. This is, however, not too surprising since the topological term was intimately related to the presence of localized sources that do not play any role in the case of heterotic compactifications.

At this point one would like to show that the potential (3.12) can be written in the standard $\mathcal{N} = 1$ form

$$V = \mathrm{e}^{\mathcal{K}} \left[g^{i\bar{\jmath}} D_i W \overline{D_{\bar{\jmath}} W} - 3|W|^2 \right] , \tag{3.13}$$

where $\mathcal{K}$ is the Kähler potential of the moduli space and $g^{i\bar{\jmath}}$ is the inverse metric following from $\mathcal{K}$. In the IIB case this followed from the fact that $e^{4A}G^+$ is harmonic on the conformally rescaled internal space, which is Calabi-Yau, and therefore one can expand it in terms of the harmonic forms on the same manifold. In the heterotic case, however, one is faced with two problems: the moduli space of the internal manifold is not known and the complex form $\mathcal{H}^+$ is neither closed nor coclosed. For what concerns the first problem one would need to obtain a better understanding of the geometric properties of the six-dimensional manifolds [37] involved in obtaining supersymmetric vacua of the heterotic theory. Unfortunately, it is difficult to understand the general structure of their moduli space. An additional source of complications is given by the fact that not all the manifolds in such classes are solutions also of the three-form Bianchi identity (2.14).

Since the $\mathcal{H}$-form is not closed nor co-closed, one cannot expand it into harmonic forms of the internal space as in the IIB case [16, 32]. The hope is that some other expansion may be performed which replaces that of [16, 32], and that from such an expansion one can read off the superpotential in an analogous way. Assuming this to be the case, a simple guess for the superpotential is given by

$$W = \int \mathcal{H} \wedge \Omega = \int \left(H + \frac{i}{2} dJ \right) \wedge \Omega \,. \tag{3.14}$$

[4]This product is positive definite: $\langle \alpha, \alpha \rangle \geq 0$ and $\langle \alpha, \alpha \rangle = 0 \Leftrightarrow \alpha = 0$.

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Note that the term proportional to $d\phi \wedge J$ in $\mathcal{H}$ has dropped out of (3.14) due to its Hodge type. A similar superpotential was also suggested in [52], through a dualization procedure of the IIB potential. Here we provide alternative arguments for justifying the guess (3.14) with some new motivations for the appearance of the extra dJ term.

It is interesting to notice that in (3.14) the two-form potential B has been combined with the almost complex structure two-form J (which is not closed in general) to give the complexified area

$$\mathcal{J} = B + \frac{i}{2} J\,. \tag{3.15}$$

In the Kähler case $d\mathcal{J} = dB$, so that at the field-strength level one does not see the explicit appearance of this complex combination. In the heterotic theory one also needs to extend the three-form field-strength to include the gauge and Lorentz Chern-Simons terms, $H = dB + \alpha' \left(\omega_L - \omega_{YM}\right)$. On the other hand, supersymmetry imposes that the introduction of H-fluxes must be accompanied by a non-vanishing dJ and these two are mutually non-local, i.e. $H \wedge dJ \neq 0$.[5] Thus it seems appropriate to introduce the combination $\mathcal{H}$ in the definition of the superpotential (3.14).

Extending further the analogy with the type-IIB case, one can ask whether the locking condition (2.24) follows from $W = \partial W = 0$. The vanishing of the superpotential $W = 0$ implies that the $(0,3)$ part of $\mathcal{H}$ has to vanish. On the other hand, in the heterotic case we do not have an explicit dilaton dependence in the complex three-form defining the superpotential, as was the case in the IIB theory (3.1). This means that one cannot obtain the vanishing of the $(3,0)$ part of $\mathcal{H}$ from the variation of the superpotential with respect to the dilaton. On the other hand, assuming that an overall radial modulus ρ exists, the scaling behaviour of the almost complex structure on ρ is given by

$$dJ = (\rho + \bar{\rho}) dJ|_{\rho+\bar{\rho}=1}\,. \tag{3.16}$$

Extremization of the superpotential[6] with respect to ρ then gives

$$dJ^{(0,3)} = 0\,. \tag{3.17}$$

By reality it follows then that

$$dJ^{(3,0)} = dJ^{(0,3)} = 0\,, \qquad H^{(3,0)} = H^{(0,3)} = 0\,. \tag{3.18}$$

It is then easy to see that this implies the vanishing of the first torsion class $\mathcal{W}_1 = 0$ in (2.11). Next, assuming that H is constant under variation of the almost complex structure moduli and that $\partial_i \Omega = k_i \Omega + \chi_i$, where χ_i are a set of $(2,1)$ forms and k_i are constants over the internal manifold, one further obtains that the $(1,2)$ part of $\mathcal{H}$ is vanishing. Altogether, one finds that most of the components of the complex three-form $\mathcal{H}$ have to vanish,

$$\mathcal{H}^{(3,0)} = \mathcal{H}^{(0,3)} = \mathcal{H}^{(1,2)} = 0\,. \tag{3.19}$$

[5] In view of the analogy with type IIB, we note that there as well it is not possible to have non-trivial supersymmetric flux solutions when only one flux system, say H_R, is turned on: if $e \times m$ is zero, then one can rotate all fluxes to be electric, but $W = e_I X^I$ is not non-trivially solvable in the e_I on a non-singular Calabi-Yau space.

[6] Note that the superpotential is nevertheless holomorphic in ρ.

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On a complex manifold, the last of these conditions yields the locking of the three-form flux onto the geometry, namely

$$\mathcal{H}^{(1,2)} = H^{(1,2)} + \frac{i}{2}\bar{\partial} J = 0\,, \tag{3.20}$$

and since H is a real form, the desired relation follows,

$$H = \frac{i}{2}\left(\partial - \bar{\partial}\right) J\,. \tag{3.21}$$

An analogous situation arises also in type-IIA compactifications with RR 2-form flux. In this case the backreaction of the fluxes induces a strong deformation of the geometry such that $\mathcal{M}_6$ will not be any more complex or Kähler. This can be seen performing [38] a mirror transformation starting from IIB models with NS 3-form flux, or lifting type IIA with 2-form flux to M-theory on a G_2 holonomy manifold (see the discussion in the appendix of [37] and [29, 43, 50]). The result is that $\mathcal{M}_6$ is characterized by non-vanishing torsion in $\mathcal{W}_2^+$. So the type-IIA space $\mathcal{M}_6$ is an almost-Kähler manifold. This space has a non-closed 3-form Ω, i.e. $d\Omega \neq 0$. As first discussed in [10] (see also the brief discussion in [22]) and further elaborated on in [36], $d\Omega$ provides a purely geometrical term to the type-IIA superpotential, a so-called NS 4-form flux, and the type-IIA superpotential then takes the form

$$W = \int \left(H_R^{(2)} \wedge J + d\Omega\right) \wedge J\,. \tag{3.22}$$

Like for the term $dJ \wedge \Omega$ in the heterotic superpotential (1.1), the deviation of $\mathcal{M}_6$ from being a Calabi-Yau space is measured in type IIA by $d\Omega \wedge J$ (note that by partial integration these two terms agree), and space-time supersymmetry links together $H_R^{(2)}$ with $d\Omega$.

4. Comments on the radial modulus and higher order α' corrections

In the quest for explicit supersymmetric solutions of the heterotic theory with fluxes the five torsion classes for SU(3) structures defined in [57] are an important tool. As shown in [37], given the information about the intrinsic torsion of the allowed manifolds, it is very easy to provide entire classes of candidate solutions by simply consulting the mathematical literature. In the same way, new classes of solutions can be constructed upon analysis of the conditions (2.8)–(2.10) [42]. However, as pointed out in [37], it does not seem possible to also solve the H Bianchi identity for a given general class as was the case for Calabi-Yau manifolds. This means that, in order to decide whether a certain manifold provides a consistent supersymmetric heterotic background, one needs to explicitly solve the modified three-form Bianchi identity (2.14) on a case by case basis.

An interesting issue which arises for these solutions is the stabilization of the overall size modulus. In all the known examples of IIB compactifications the resulting effective four-dimensional models were of no-scale type [8, 19, 23, 25, 31, 35, 38] and the overall size modulus remained free even after the introduction of fluxes. This cannot happen in the heterotic compactifications because of the modified Bianchi identity for the three-form [55, 46]. As we will now show, the various terms appearing in equation (2.14) generically scale

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differently with respect to this parameter and therefore it needs to be fixed in order to obtain a consistent solution. Given a generic metric with overall size modulus $t \equiv \sqrt{\rho + \bar{\rho}}$, i.e. $ds^2 = t^2(\ldots)$, the two-form associated to the complex structure J scales with $J \to t^2 J$. Since for supersymmetric backgrounds the three-form H is related to J by (2.24), its exterior derivative scales accordingly, i.e.

$$dH \to t^2 dH\,. \tag{4.1}$$

On the other hand it is straightforward to check the following scaling relations for the curvature,

$$R^a{}_{bcd} \sim t^0\,, \qquad \mathrm{tr}\, R \wedge R \sim t^0\,. \tag{4.2}$$

Consequently, the generic solution of the H Bianchi identity (2.14) imposes a second order equation in the parameter t, which will generically result in t becoming fixed.

This feature raises the question of the validity of the supergravity approximation we have used. The quadratic equation fixing the value of the overall size modulus depends on α' and generically leads to compactification manifolds with a characteristic radius of $\sqrt{\alpha'}$. This, of course, implies a breakdown of the supergravity approximation and therefore one should not trust the generic solution found in this way. A possible way out [52] is to allow a dependence of t on the flux density, which enters in the determination of the background geometry. For instance, the authors of [52] argue that the overall size modulus of their solution is given by $t^6 = \alpha'|f|^2$, where f is the flux density. Since in principle f can be taken to be arbitrarily large, the overall size of the manifold can still be large. As appealing as this feature may be, it does unfortunately not guarantee that the supergravity approximation can be trusted, as we will now show for a special class of manifolds.

An interesting subclass of solutions of the geometrical constraints (2.8)–(2.10) is given by manifolds which are products of tori and twisted tori (included in the class of the nilmanifolds in [37]) or by torus fibration of Calabi-Yau two-folds, where the fiber is twisted [33, 37, 42, 45, 46, 52]. A simple instance of such spaces is given by a metric of the form

$$ds^2 = t^2 \left[ds_4^2(z,\bar{z}) + \left(dv + N \sum_j f^j(z,\bar{z}) dz^j \right) \left(d\bar{v} + N \sum_j \bar{f}^j(z,\bar{z}) d\bar{z}^j \right) \right], \tag{4.3}$$

where we use dimensionless coordinates. In this equation the twist in the metric is associated to a deformation parameter N, which is related to the flux. Using (3.21) one finds that

$$H \sim N t^2 \sum_k df^k \wedge dz^k \wedge \left(d\bar{v} + N \sum_j \bar{f}^j d\bar{z}^j \right), \tag{4.4}$$

from which it follows that

$$dH \sim N^2 t^2 \sum_{j,k} df^k \wedge d\bar{f}^j \wedge dz^k \wedge d\bar{z}^j\,. \tag{4.5}$$

An analogous computation for the $\mathrm{tr}\, R \wedge R$ terms yields

$$\mathrm{tr}\, R \wedge R \sim t^0 N^4 \sum_{j,k} df^k \wedge d\bar{f}^{\bar{j}} \wedge dz^k \wedge d\bar{z}^{\bar{j}} \,. \tag{4.6}$$

Inserting this into the H-Bianchi identity (2.14) shows that the overall size is fixed not only in terms of α', but also in terms of the flux as

$$t \sim N\sqrt{\alpha'} \,. \tag{4.7}$$

Therefore a large value of N results in a large value of t. A large value of t, however, does not guarantee that the supergravity approximation can be trusted. Indeed, when evaluating the Ricci scalar on the solution, one finds that it is of order $1/\alpha'$, i.e.

$$R \sim t^{-2} N^2 \sim \frac{1}{\alpha'} \,, \tag{4.8}$$

where we used (4.7). Thus not only the size but also the curvature scales with the flux. Since each term in the effective action, when evaluated on the solution, is of order $1/\alpha'$, one should retain the entire tower of α' terms in the action. The reason why this happens for twisted tori metrics is because the $\alpha' \to 0$ limit is singular for these backgrounds.

One may thus be tempted to instead consider manifolds which admit a smooth $\alpha' \to 0$ limit. However, in this case the resulting geometry is simply a deformation of the underlying Calabi-Yau manifold, and it has been shown that there is no moduli fixing [53] at all. Another possibility for achieving a consistent moduli fixing in heterotic compactifications with fluxes is of course to consider solutions which still do not admit a smooth $\alpha' \to 0$ limit, but which do not fall into the class described above.

To conclude this discussion, let us try to understand how the manifolds described by the constraints (2.8)–(2.10) will be modified by higher order terms in α'.

The only way to obtain the exact background geometry for flux compactifications is by a detailed knowledge of the corresponding string σ-model. As a first approximation, one considers the related supergravity theory for which the α' corrections can be computed by supersymmetry requirement. These terms follow from the modification of the H-Bianchi identity due to the Green-Schwarz anomaly cancellation mechanism. This modification breaks the supersymmetry of the standard type-I supergravity coupled to super Yang-Mills in ten dimensions. To restore it one has to add new terms both to the supersymmetry rules and to the action. This can be done either order by order in α' [66, 68], or by directly including all order corrections giving rise to non-linear curvature parameterizations [64, 65, 67]. An important drawback of both these approaches is the existence of ambiguities in the resulting theories which can only be fixed by the string computations. For instance, it is known that at the quartic level the effective action obtained from loop σ-model computations contains R^4 invariants whose coefficient goes with $\zeta(3)$. This term does not appear explicitly in any of the supergravity computations as it is not required by supersymmetry. The hope is that the corrections obtained in this way may be a consistent truncation of the full theory. In any case, they are indicative of the final structure of the

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supersymmetry transformation rules and are helpful in providing some understanding of what will happen to the torsional constraints which were derived to first order in α'.

The supersymmetry transformations of the gravitino and dilatino do not depend anylonger on the same combination of the heterotic three-form H, as was the case to $O(\alpha')$, but on two different tensors,

$$\delta\psi_M = \nabla_M\epsilon - \frac{1}{36}\Gamma_M\Gamma^{NPQ}T_{NPQ}\epsilon\,, \tag{4.9}$$

$$\delta\lambda = -2\nabla\!\!\!/\phi\epsilon + \Gamma^{MNP}Z_{MNP}\epsilon\,. \tag{4.10}$$

The torsion tensor T also appear in the definition of the H Bianchi identity,

$$dH = \alpha'\left(\mathrm{tr}\,\widetilde{R}\wedge\widetilde{R} - \mathrm{tr}\,F\wedge F\right), \tag{4.11}$$

since the definition of the generalized curvature $\widetilde{R}$ includes the torsion T. The difference with the order zero analysis is that now T is not simply identified with (a conformally rescaled) H, but it receives non-linear higher order corrections,

$$e^{\frac{4}{3}\phi}T_{MNP} = -3H_{MNP} + \alpha' W_{MNP}\,. \tag{4.12}$$

The same happens to the Z tensor which is defined as

$$e^{\frac{4}{3}\phi}Z_{MNP} = \frac{1}{6}T_{MNP} + \alpha' X_{MNP}\,. \tag{4.13}$$

Here both W and X are three-forms depending non-linearly on T and $\widetilde{R}$ (The relations are discussed in a certain Weyl frame, but the outcome is frame independent). Obviously it is almost impossible to discuss the resulting geometrical constraints, but it is also clear that the relations (2.8)–(2.10) will be modified. In particular it is not guaranteed that the internal geometry will stay complex, since this condition was crucially dependent on the fact that both the gravitino and dilatino supersymmetry variation depended on the same three-form H, whereas here we have two different tensors. Moreover, even if the geometry still has SU(3) holonomy with respect to the generalized connection with torsion T, we do not expect that (2.24) will persist to higher orders in α'. The derivation of this locking condition depended once more on the peculiar structure of the supersymmetry transformations to lowest order in α', with the same type of structure entering in the gravitino and dilatino rules. The only relation which will remain valid in the presence of higher order α' corrections is the identification of the Nijenhuis tensor with the $(3,0)+(0,3)$ parts of the torsion tensor,

$$N_{mnp} = 4\,T_{mnp}^{(3,0)+(0,3)}\,. \tag{4.14}$$

Acknowledgments

We would like to thank M. de Roo, K. Lechner, I. Pesando and N. Prezas for valuable discussions as well as K. Dasgupta, B. Körs, A. Krause, D. Martelli, G. Papadopoulos and D. Waldram for useful correspondence. The work of G.L.C. is supported by the DFG. Work supported in part by the European Community's Human Potential Programme under contract HPRN–CT–2000–00131 Quantum Spacetime.

A. Some useful identities

The definition for the Ricci scalar $\widetilde{R}^a{}_{bac}g^{bc}$ follows from the Riemann curvature,

$$\widetilde{R}^a{}_{bcd} = 2\partial_{[c}\widetilde{\Gamma}^a{}_{d]b} + 2\widetilde{\Gamma}^a{}_{[c|m}\widetilde{\Gamma}^m{}_{d]b} \tag{A.1}$$

computed from the connection with torsion T,

$$\widetilde{\Gamma}^a{}_{bc} = \Gamma^a{}_{bc} + T^a{}_{bc}\,. \tag{A.2}$$

Moreover the covariant derivative is defined as

$$\widetilde{\nabla}_a J_b{}^c = \partial_a J_b{}^c - \widetilde{\Gamma}^s{}_{ab}J_s{}^c + \widetilde{\Gamma}^c{}_{as}J_b{}^s\,. \tag{A.3}$$

For a totally antisymmetric torsion tensor it also follows that

$$\widetilde{R}_{ij} = R_{ij} - T_{ilm}T_j{}^{lm} - \nabla^k T_{kij}\,, \tag{A.4}$$

and

$$\widetilde{R} = R - T_{abc}T^{abc}\,. \tag{A.5}$$

The torsion T for the heterotic theory will be $\pm H$. The associated spin connection with torsion is then given by $\widetilde{\omega}_m^{ab} = \omega_m^{ab} \mp H_m^{ab}$, and this is how one distinguishes the corresponding connections $\nabla^{\mp}$ and curvatures $R^{\mp}$.

In the paper we considered almost-complex riemannian manifolds admitting an SU(3) structure. Using the almost complex structure, one can define the projector tensors

$$P_a{}^b = \frac{1}{2}\left(\delta_a^b - \mathrm{i}J_a{}^b\right), \quad Q_a{}^b = \frac{1}{2}\left(\delta_a^b + \mathrm{i}J_a{}^b\right), \tag{A.6}$$

which define the $(1,0)$ and $(0,1)$ projected components of a generic form. For instance, we can split the components of a three-form, say T, as follows,

$$\begin{aligned} T^{(3,0)}_{abc} &= P_a{}^d P_b{}^e P_c{}^f T_{def}\,,\\ T^{(2,1)}_{abc} &= 3P_a{}^d P_b{}^e Q_c{}^f T_{def}\,,\\ T^{(1,2)}_{abc} &= 3P_a{}^d Q_b{}^e Q_c{}^f T_{def}\,,\\ T^{(0,3)}_{abc} &= Q_a{}^d Q_b{}^e Q_c{}^f T_{def}\,, \end{aligned} \tag{A.7}$$

and the total decomposition of the three-form T according to type is unique

$$T_{abc} = T^{(3,0)}_{abc} + T^{(2,1)}_{abc} + T^{(1,2)}_{abc} + T^{(0,3)}_{abc}\,. \tag{A.8}$$

The same decomposition can be done for any other generic p-form. Exterior differentiation will preserve the type only for complex manifolds.

Some very useful relations which appear when one considers only "real" combinations are

$$T^{(3,0)}_{abc} + T^{(0,3)}_{abc} = \frac{1}{4}\left(T_{abc} - 3J_{[a}{}^p J_b{}^q T_{c]pq}\right), \tag{A.9}$$

$$T^{(2,1)}_{abc} + T^{(1,2)}_{abc} = \frac{3}{4}\left(T_{abc} + J_{[a}{}^p J_b{}^q T_{c]pq}\right). \tag{A.10}$$

If one in particular considers the square of (A.9) a further simplification arises

$$4\left(T^{(3,0)}_{abc}+T^{(0,3)}_{abc}\right)^2 = T_{abc}T^{abc} - 3T_{abc}J^{aq}J^{bs}T_{qs}{}^{c}\,. \tag{A.11}$$

and hence

$$\begin{aligned}4\int\star(T^{(3,0)}+T^{(0,3)})\wedge(T^{(3,0)}+T^{(0,3)}) &= \frac{1}{6}\int d^6y\,\sqrt{g}\left(T_{abc}T^{abc}-3T_{abc}J^{aq}J^{bs}T_{qs}{}^{c}\right)\\ &= \int\star T\wedge T-\frac{1}{2}\int d^6y\,\sqrt{g}\,T_{abc}J^{aq}J^{bs}T_{qs}{}^{c}\,, \end{aligned}\tag{A.12}$$

which is very useful in identifying some of the expressions appearing in the rewriting of the action in terms of BPS-like squares.

Imaginary (anti-)selfduality is defined according to

$$T^{\pm} = \frac{1}{2}\left(T\mp i\star T\right), \tag{A.13}$$

$$\star T^{\pm} = \pm iT^{\pm}\,. \tag{A.14}$$

The anomaly cancellation implies a modification of the Bianchi identity for the three-form H,

$$dH = \alpha'\left(p_1(F)-p_1(R)\right), \tag{A.15}$$

where $p_1(F)$ and $p_1(R)$ are the first Pontrjagin forms for the gauge and tangent bundles respectively. In terms of the Riemann and Yang-Mills curvatures this equation reads

$$dH = \alpha'\left(\mathrm{tr}\,\widetilde{R}\wedge\widetilde{R}-\mathrm{tr}\,F\wedge F\right), \tag{A.16}$$

where the trace on the gauge indices (and similarly for the generalized Riemann curvature $\widetilde{R}$) is taken as

$$\mathrm{tr}\,(F\wedge F) = F^{ab}\wedge F_{ab}\,, \tag{A.17}$$

and

$$F^{ab} = F^I(T^I)^{ab}\,, \qquad \text{with}\ \mathrm{tr}\left(T^IT^J\right)=\delta^{IJ}\,. \tag{A.18}$$

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